NSFS 2011, Peder Kock, Lund University 2011

A real-time statistical alarm method for mobile gamma spectrometry: combining binomial counting with a goodness-of-fit

2011-09-08

Outline

- Decision levels / evaluation
- Proposed method
- Three reference methods
- Results and comparisons from Monte Carlo simulations

Decision levels

- Inference about analyte activity
- Predefined level of confidence – α
- Actual false alarm rate – α'
- Challenge for low-level counting (μ ≤ 5)

Paired blanks technique

- Measure a blank sample twice:
  1. ”sample”
  2. ”background”
- All indications of analyte activity in sample are false – α’ given

Proposed method

- Reliable false positive rate
  - Validate α’
- Account for as much information in sample as possible
  - Spectral distribution
- Real time method (mobile γ-spectrometry)
  - Calculations done before next measurement

Proposed method – cont'd

- Two channels Poisson probability:

$$P(\mathbf{Z}) = \prod_{i=1}^{2} \frac{e^{-\mu_i}{\mu_i}^{x_i}}{x_i!}\frac{e^{-\lambda_i}{\lambda_i}^{y_i}}{y_i!}$$

- *P(**Z**)* given some total count N_t:

$$P(\mathbf{Z}|N_t) = \binom{N_t}{x_1 x_2 y_1 y_2} {\pi_1}^{x_1}{\pi_2}^{x_2}{\pi_3}^{y_1}{\pi_4}^{y_2}$$

where

$$\pi_i = {}^{\mu_i}/_{\mu_t}$$

Proposed method – cont'd

- Extending to *k* channels
- Do not separate sample and background

$$P(\boldsymbol{a}|\boldsymbol{\pi}, N_t) = N_t! \prod_{i=1}^{k} \frac{{\pi_i}^{a_i}}{a_i!}$$

For any parameter vector $\boldsymbol{\pi} = (\pi_1, \dots, \pi_k), \sum \pi_i = 1$

and pulse distribution $\boldsymbol{a} = (a_1, \dots, a_k)$.

Decision levels (1/2)

Sumerling & Darby (1981):

$$P_{SD} = P(Y \geq N_s | N_t) = \sum_{i=N_s}^{N_t} \binom{N_t}{i} \pi_1{}^i (1 - \pi_1)^{N_t - i} \leq \alpha$$

Sumerling, T.J. and Darby, S.C. (1981).
Statistical aspects of the interpretation of counting experiments designed to detect low levels of radioactivity.
National Radiological Protection Board (NRPB); NRPB-R113, Chilton, UK

Currie (1968):

$$DL(N_b, \alpha) = k_\alpha \sqrt{2N_b}$$

Currie, L.A. (1968).
Limits from the qualitative detection and quantitative determination.
Anal. Chem., 40(3); pp. 586-593.

Decision levels (2/2)

Stapleton's method:

$$z_{stapleton} = 2\,\frac{\sqrt{\dfrac{N_s + d}{t_s}} - \sqrt{\dfrac{N_b + d}{t_b}}}{\sqrt{\dfrac{1}{t_s} + \dfrac{1}{t_b}}}$$

Strom, D.J. and MacLellan, J.A. (2001).
Evaluation of eight decision rules for low-level radioactivity counting.
Health Phys., 81(1); pp. 27-34

Proposed method cont'd

- As shown by Strom and MacLellan (2001), the S&D method always overestimate *P(**Z**)*

$$\alpha'_{SD} < \alpha$$

- Modify S&D by multiplying with p-value of pulse configuration - κ

$$\kappa \cdot P_{SD} \leq \alpha$$

P-value of multinomial

- Naively traversing all combinations

$$a_1, \ldots, a_k \leq y$$

can be computationally very expensive

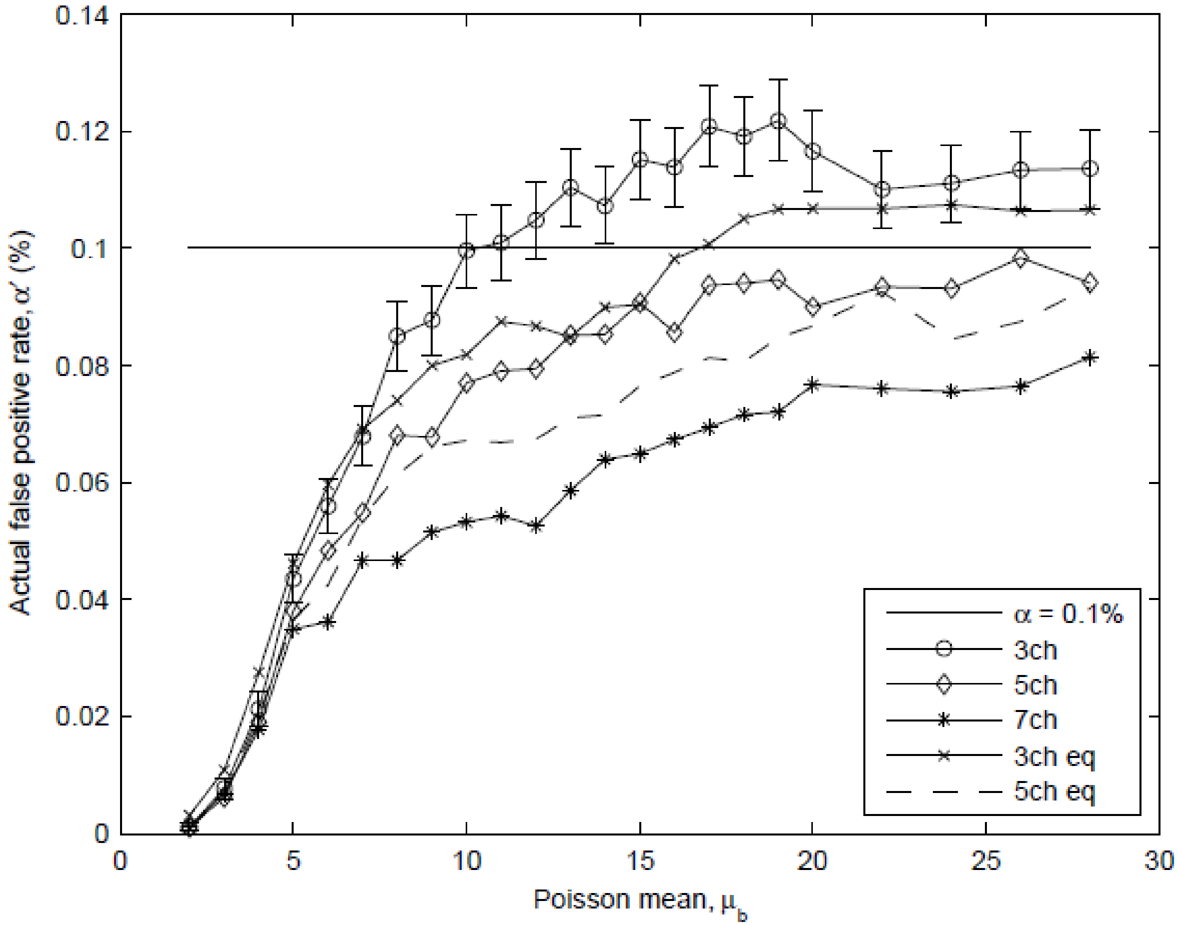
- Keich and Nagarajan (2006) approximation using FFT and lattice expansion

Keich, U. and Nagarajan, N. (2006).
A fast and numerically robust method for exact multinomial goodness-of-fit test.
J. Comput. Graph. Stat., 15(4); pp. 779-802.

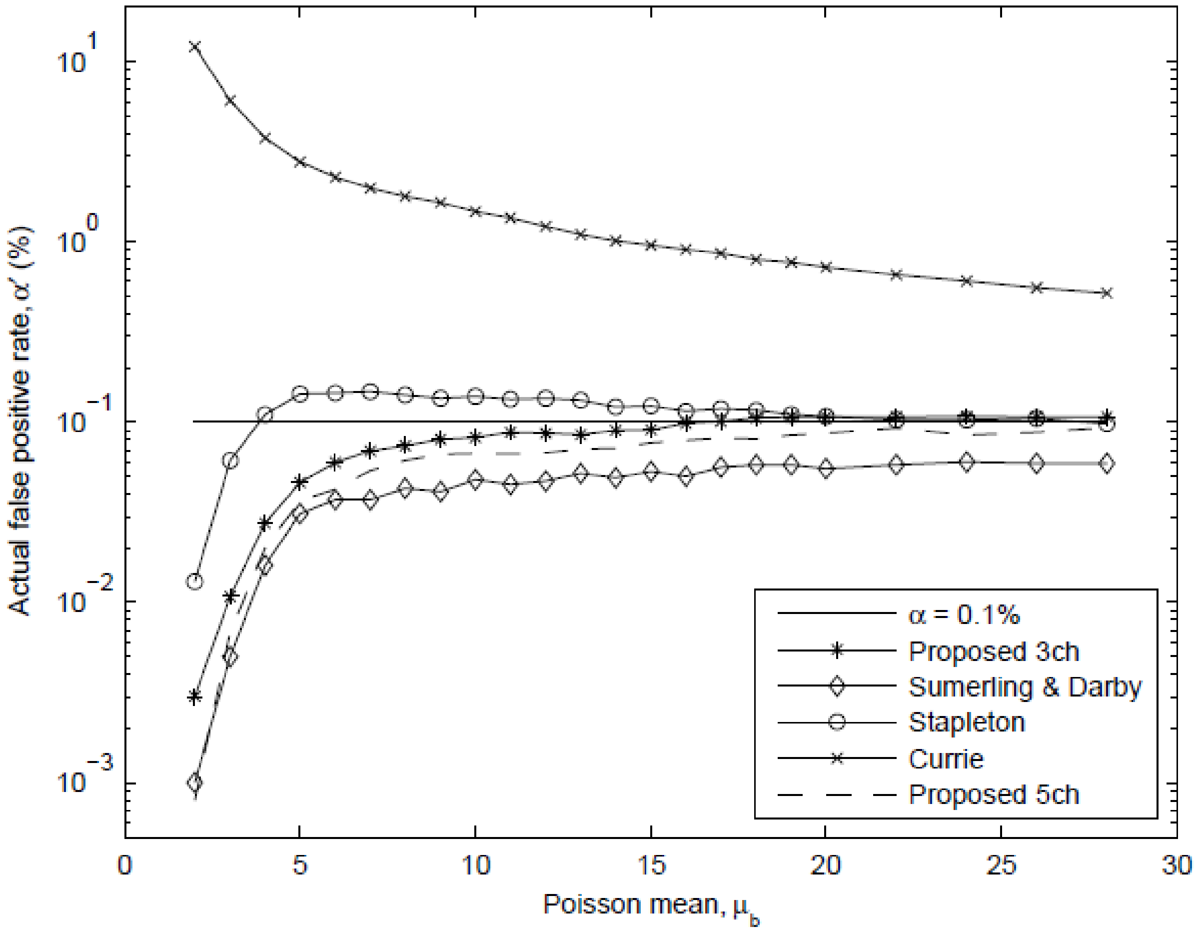
Computer code

- Keich and Nagarajan (2006)
  - BagFFT code (C++ sources)
- False positives Monte Carlo simulations:
  - Perl, C
- True positives Monte Carlo simulations:
  - Small cluster (< 10 cores), C#, Perl

Results – proposed method α’

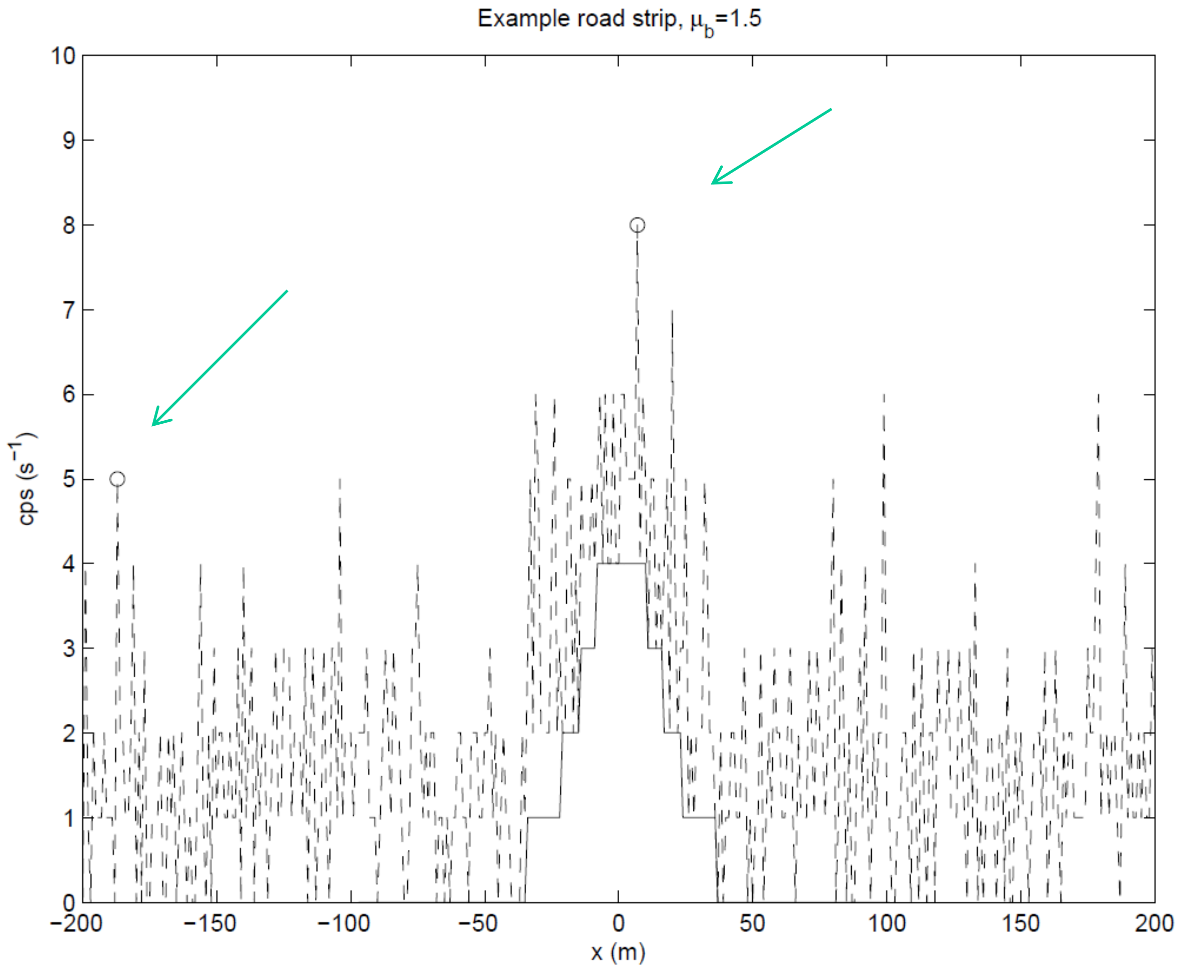


Results – α′



Actual false positive rate, α′ (%)
Poisson mean, μ_b
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
0
5
10
15
20
25
30
α = 0.1%
Proposed 3ch
Sumerling & Darby
Stapleton
Currie
Proposed 5ch

True positives simulations



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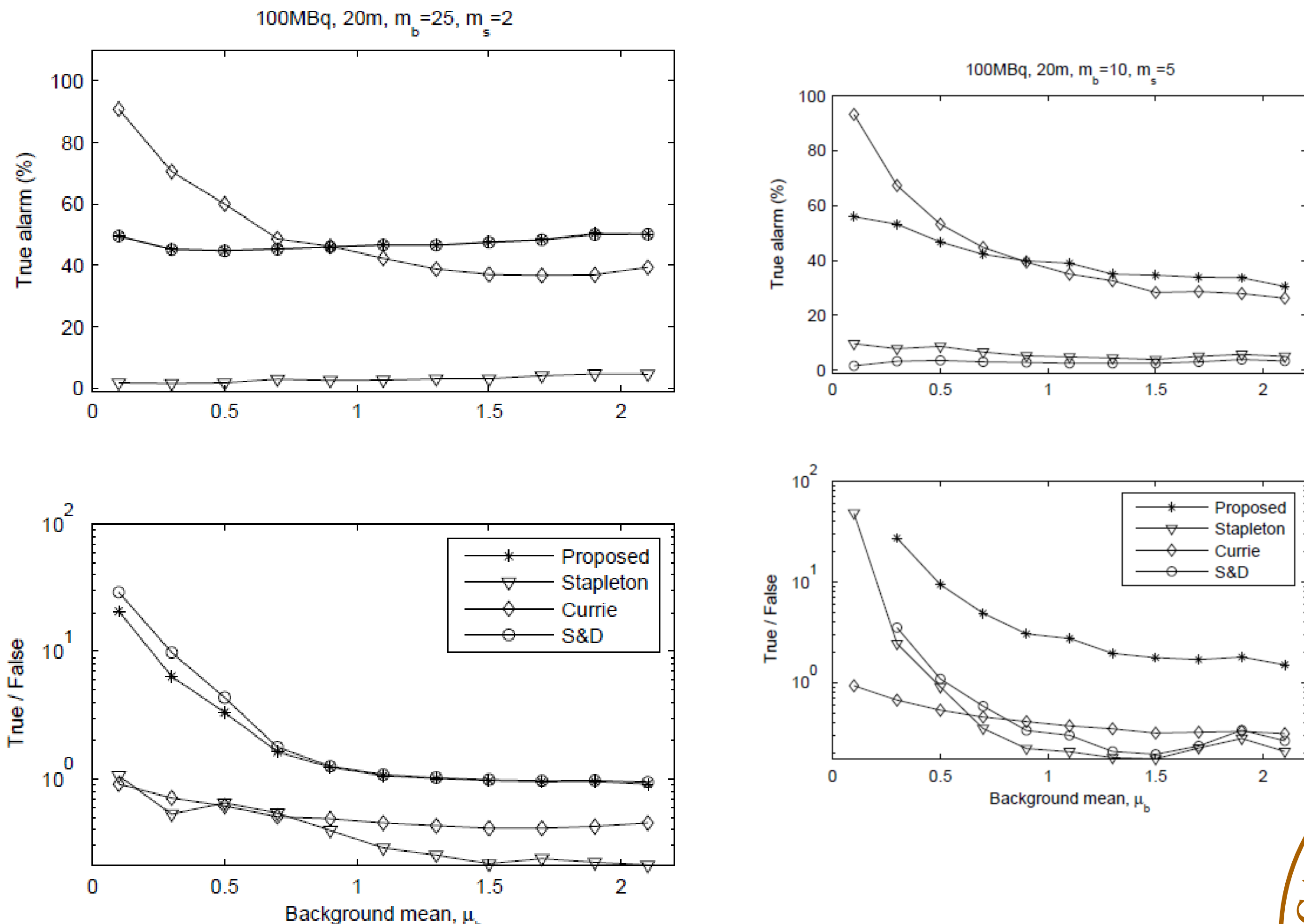
Results – true positives (1/2)



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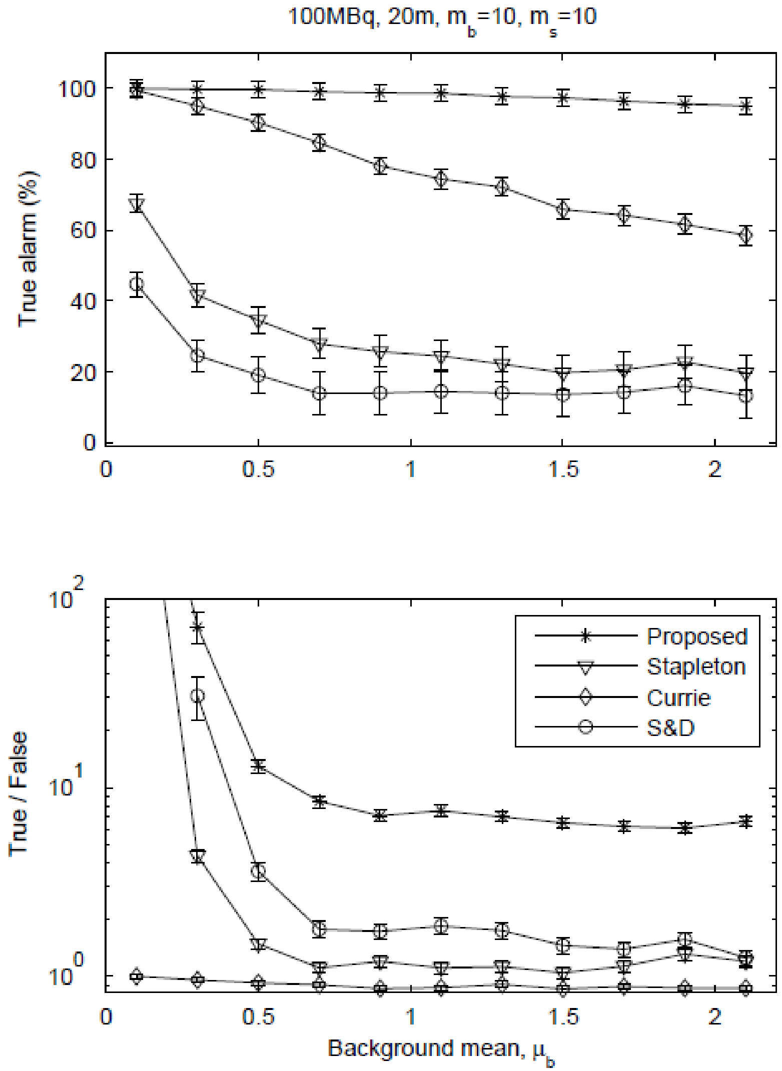
Results – true positives (2/2)



Conclusions

- Feasible to account for spectral information
  - …down to some μ
- 3-5 channels seems to be optimal
- Acceptable α', good sensitivity
- Flexible and fast method
  - <1 s computing time even at intermediate μ's

Thank you for your attention!

Questions?

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